ECS315 Quiz 2 Sol

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Consider a random experiment in which you roll a 20-sided fair dice.

$(\Omega = \{1, 2, 3, \ldots, 20\})$

Let $X(\omega) = \omega$ $Z(\omega) = |\omega - 5| - 3$

$Y(\omega) = (\omega - 5)^2$

① Find $P[X = 5]$

$X(\omega) = 5$ iff $\omega = 5$

So, $P[X=5] = P(\{5\}) = \frac{1}{20}$

② Find $P[Y = 16]$

$Y(\omega) = 16$ iff $(\omega - 5)^2 = 16$

$\omega - 5 = \pm 4$

$\omega = 5 \pm 4 = 1$ or 9

So, $P[Y = 16] = P(\{1, 9\}) = \frac{2}{20} = \frac{1}{10}$

③ Find $P[Y > 10]$

$Y(\omega) > 10$ iff $(\omega - 5)^2 > 10$

Here, we plug-in $\omega = 1, 2, \ldots, 20$ one-by-one and see that $\omega = 1, 9, 10, 11, \ldots, 20$ satisfy the condition.

So, $P[Y > 10] = P(\{1, 9, 10, 11, \ldots, 20\}) = \frac{13}{20}$

ω	$(\omega-5)^2$	>10?
1	16	✓
2	9	×
3	4	×
4	1	×
5	0	×
6	1	×
7	4	×
8	9	×
9	16	✓
10	25	✓
⋮	↓ increasing so, >10	⋮

Alternatively, you may remember that $(\omega - 5)^2 > 10$ iff $\omega - 5 > \sqrt{10}$ or $\omega - 5 < -\sqrt{10}$

$\Updownarrow$ $\Updownarrow$

$\omega > \underbrace{5 + \sqrt{10}}_{= 8.1623}$ $\omega < \underbrace{5 - \sqrt{10}}_{1.8377}$

$\Updownarrow$ $\Updownarrow$

$\omega = 9, 10, \ldots, 20$ $\omega = 1$

④ Find $P[Z > 10]$

$Z(\omega) > 10$ iff $|\omega - 5| - 3 > 10$

$|\omega - 5| > 13$

Here, we plug-in $\omega = 1, 2, \ldots, 20$ one-by-one and see that $\omega = 19, 20$ satisfy the condition.

So $P[Z > 10] = P(\{19, 20\}) = \frac{2}{20} = \frac{1}{10}$

ω	$\lvert\omega-5\rvert$	>13?	$8 < \cdot < 13$?
1	4	×	×
2	3	×	×
3	2	×	×
4	1	×	×
5	0	×	×
6	1	×	×
7	2	×	×
8	3	×	×
9	4	×	×
10	5	×	×
11	6	×	×
12	7	×	×

8	3	×	×
9	4	×	×
10	5	×	×
11	6	×	×
12	7	×	×
13	8	×	×
14	9	×	✓
15	10	×	✓
16	11	×	✓
17	12	×	✓
18	13	×	×
19	14	✓	×
20	15	✓	×

condition.

So, $P[Z>10] = P(\{19, 20\}) = \frac{2}{20} = \frac{1}{10}$

Alternatively, you may remember that $|\omega-5|>13$ iff $\omega-5>13$ or $\omega-5<-13$

$\omega-5>13 \Leftrightarrow \omega>18 \Leftrightarrow \omega=19,20$

$\omega-5<-13 \Leftrightarrow \omega<-8$ ← No ω satisfies this

⑤ Find $P[5<Z<10]$

$5 < Z(\omega) < 10$ iff $5 < |\omega-5|-3 < 10$

$8 < |\omega-5| < 13$

Here, we plug-in $\omega = 1, 2, \ldots, 20$ one-by-one and see that $\omega = 14, 15, 16, 17$ satisfy the condition.

So, $P[5<Z<10] = P(\{14, 15, 16, 17\}) = \frac{4}{20} = \frac{1}{5}$

Alternatively, you may remember that $|\omega-5|>8$ iff $\omega-5>8$ or $\omega-5<-8$

$\omega-5>8 \Leftrightarrow \omega>13$

$\omega-5<-8 \Leftrightarrow \omega<-3$ ← No ω satisfies this

$|\omega-5|<13$ iff $-13<\omega-5<13 \Leftrightarrow -8<\omega<18$

Here, ω need to satisfy both these condition. So, $13<\omega<18 \Leftrightarrow \omega = 14, 15, 16, 17$