

Consider a random experiment in which you roll a 20-sided fair dice.

$$(\Omega = \{1, 2, 3, \dots, 20\})$$

Let $X(\omega) = \omega$ $Z(\omega) = |\omega - 5| - 3$
 $Y(\omega) = (\omega - 5)^2$

① Find $P[X = 5]$

$X(\omega) = 5$ iff $\omega = 5$
 So, $P[X = 5] = P(\{5\}) = \frac{1}{20}$

② Find $P[Y = 16]$

$Y(\omega) = 16$ iff $(\omega - 5)^2 = 16$
 $\omega - 5 = \pm 4$
 $\omega = 5 \pm 4 = 1 \text{ or } 9$

So, $P[Y = 16] = P(\{1, 9\}) = \frac{2}{20} = \frac{1}{10}$

③ Find $P[Y > 10]$

$Y(\omega) > 10$ iff $(\omega - 5)^2 > 10$

Here, we plug-in $\omega = 1, 3, \dots, 20$ one-by-one and see that $\omega = 1, 9, 10, 11, \dots, 20$ satisfy the condition.

So, $P[Y > 10] = P(\{1, 9, 10, 11, \dots, 20\}) = \frac{13}{20}$

Alternatively, you may remember that $(\omega - 5)^2 > 10$ iff $\omega - 5 > \sqrt{10}$ or $\omega - 5 < -\sqrt{10}$
 $\omega > 5 + \sqrt{10} = 8.1623$ $\omega < 5 - \sqrt{10} = 1.8377$
 $\omega = 9, 10, \dots, 20$ $\omega = 1$

| ω | $(\omega - 5)^2$ | $> 10?$ |
|----------|------------------|---------|
| 1 | 16 | ✓ |
| 2 | 9 | X |
| 3 | 4 | X |
| 4 | 1 | X |
| 5 | 0 | X |
| 6 | 1 | X |
| 7 | 4 | X |
| 8 | 9 | X |
| 9 | 16 | ✓ |
| 10 | 25 | ✓ |
| ... | ... | ... |

increasing
so, > 10

④ Find $P[Z > 10]$

$Z(\omega) > 10$ iff $|\omega - 5| - 3 > 10$
 $|\omega - 5| > 13$

Here, we plug-in $\omega = 1, 3, \dots, 20$ one-by-one and see that $\omega = 19, 20$ satisfy the condition.

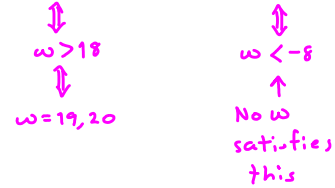
So $P[Z > 10] = P(\{19, 20\}) = \frac{2}{20} = \frac{1}{10}$

| ω | $ \omega - 5 $ | $> 13?$ | $8 < \cdot < 13?$ |
|----------|----------------|---------|-------------------|
| 1 | 4 | X | X |
| 2 | 3 | X | X |
| 3 | 2 | X | X |
| 4 | 1 | X | X |
| 5 | 0 | X | X |
| 6 | 1 | X | X |
| 7 | 2 | X | X |
| 8 | 3 | X | X |
| 9 | 4 | X | X |
| 10 | 5 | X | X |
| 11 | 6 | X | X |
| 12 | 7 | X | X |
| 13 | 8 | X | X |
| 14 | 9 | X | X |
| 15 | 10 | X | X |
| 16 | 11 | X | X |
| 17 | 12 | X | X |
| 18 | 13 | X | X |
| 19 | 14 | ✓ | ✓ |
| 20 | 15 | ✓ | ✓ |

condition.

$$\text{So, } P[Z > 10] = P(\{19, 20\}) = \frac{2}{20} = \frac{1}{10}$$

Alternatively, you may remember that $|w-5| > 13$ iff $w-5 > 13$ or $w-5 < -13$



| | | | |
|----|----|---|---|
| 8 | 3 | | |
| 9 | 4 | | |
| 10 | 5 | | |
| 11 | 6 | X | X |
| 12 | 7 | X | X |
| 13 | 8 | X | X |
| 14 | 9 | X | X |
| 15 | 10 | X | X |
| 16 | 11 | X | X |
| 17 | 12 | X | X |
| 18 | 13 | X | X |
| 19 | 14 | ✓ | ✓ |
| 20 | 15 | ✓ | ✓ |

⑤ Find $P[5 < Z < 10]$

$$5 < Z(w) < 10 \quad \text{iff} \quad 5 < |w-5| - 3 < 10$$

$$8 < |w-5| < 13$$

Here, we plug-in $w = 1, 3, \dots, 20$ one-by-one and see that $w = 14, 15, 16, 17$ satisfy the condition.

$$\text{So, } P[5 < Z < 10] = P(\{14, 15, 16, 17\}) = \frac{4}{20} = \frac{1}{5}$$

Alternatively, you may remember that $|w-5| > 8$ iff $w-5 > 8$ or $w-5 < -8$



$$|w-5| < 13 \quad \text{iff} \quad -13 < w-5 < 13$$

$$\Downarrow$$

$$-8 < w < 18$$

Here, w need to satisfy both these condition. So, $13 < w < 18$

$$\Downarrow$$

$$w = 14, 15, 16, 17$$